JOHANNES GUTENBERG
UNIVERSITÄT MAINZ

Online Density Estimation of Heterogeneous Data Streams in Higher Dimensions

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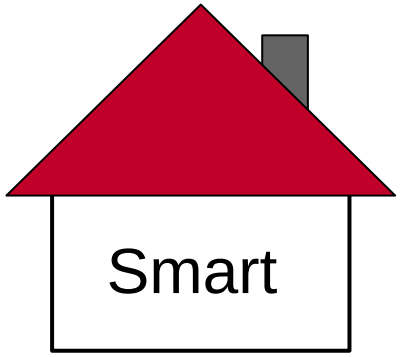
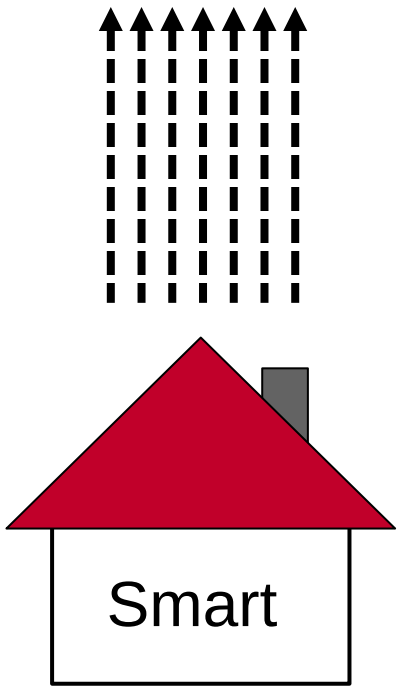
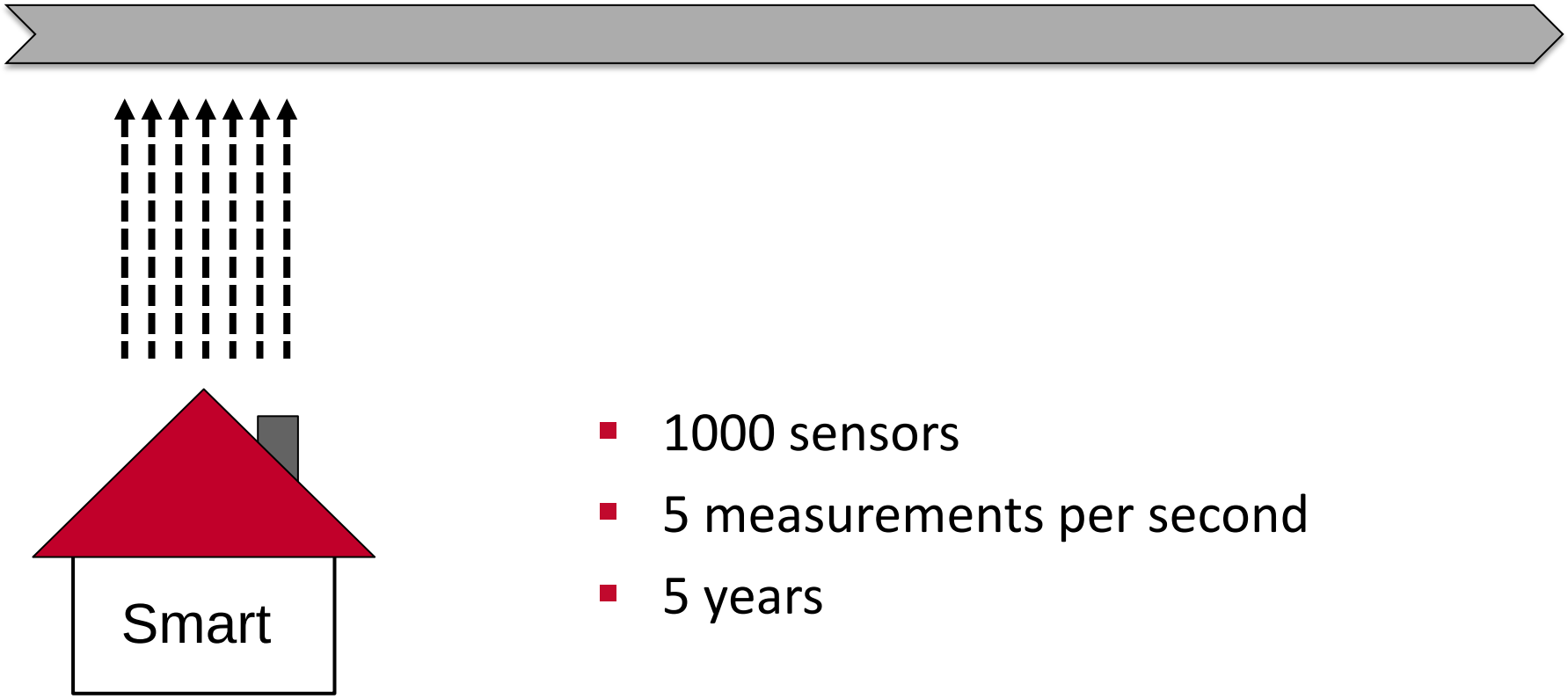

Smart


Smart

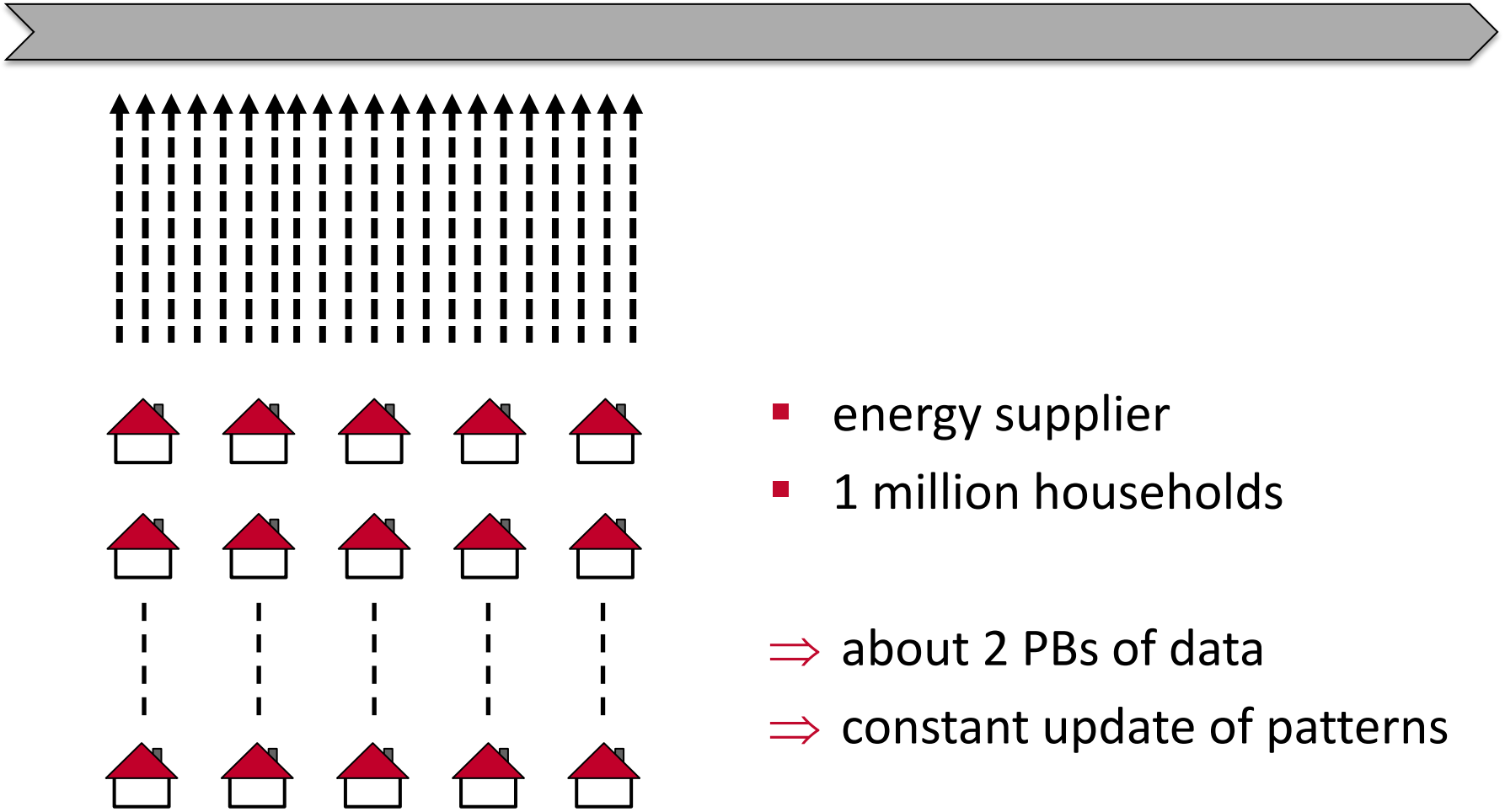

Smart



- 1000 sensors
- 5 measurements per second
- 5 years

$\Rightarrow$ more than 2 billion measurements

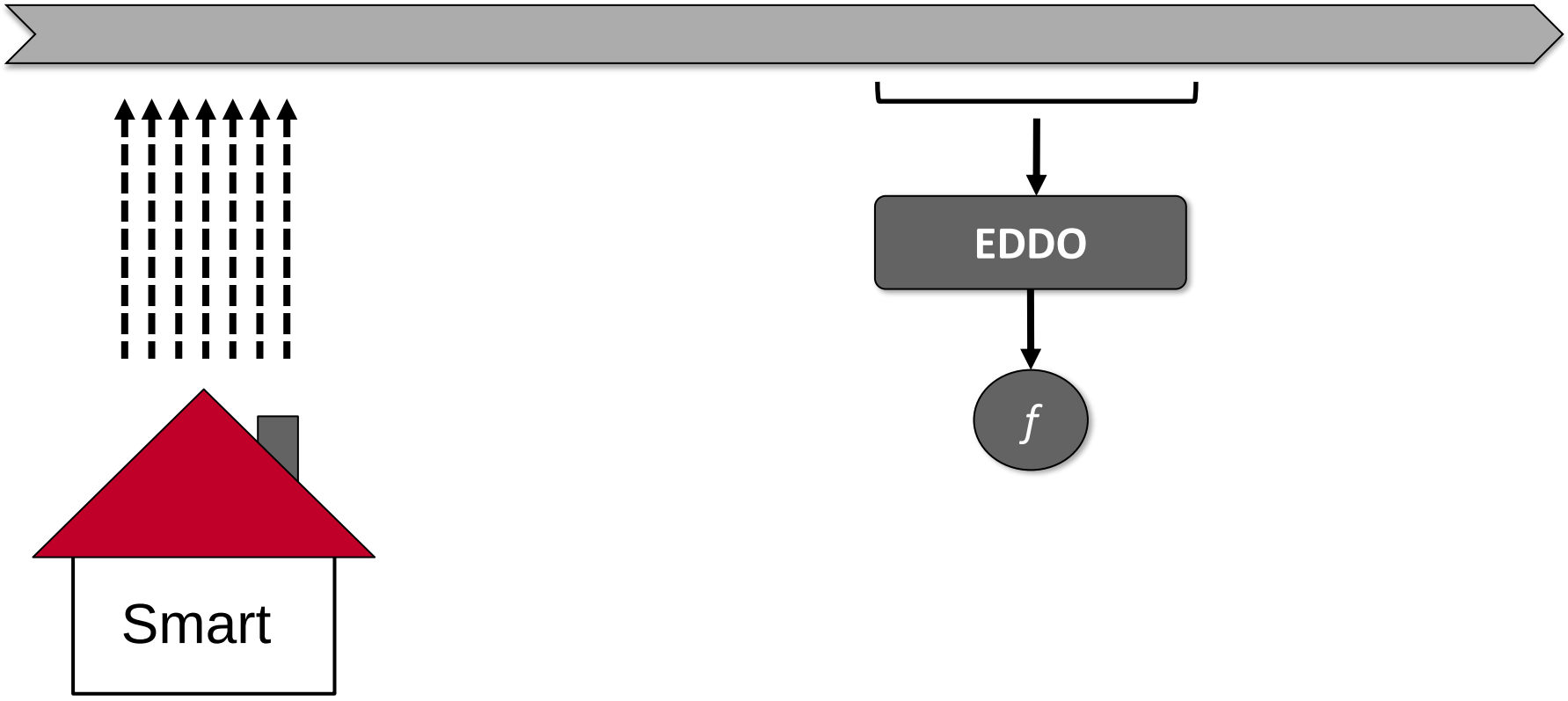
$\Rightarrow$ about 2 GBs of data

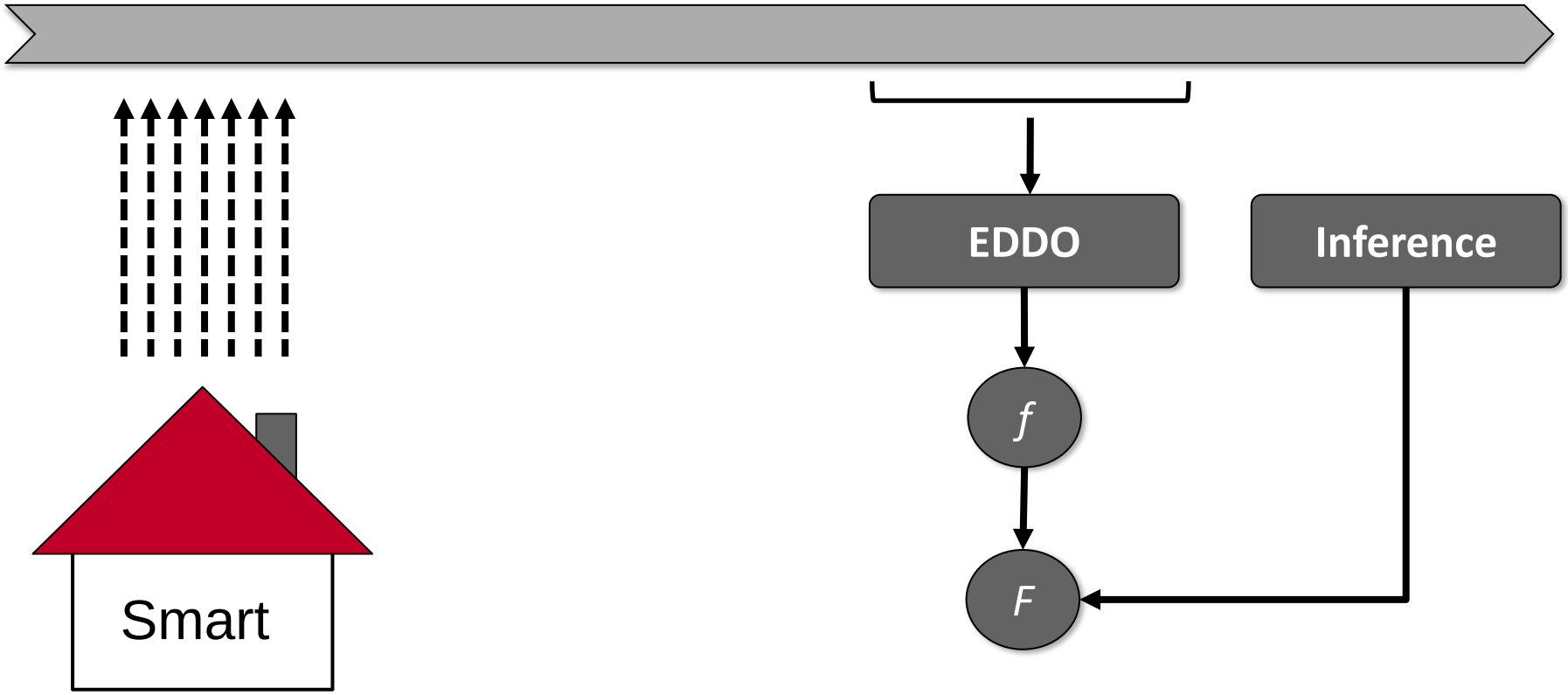


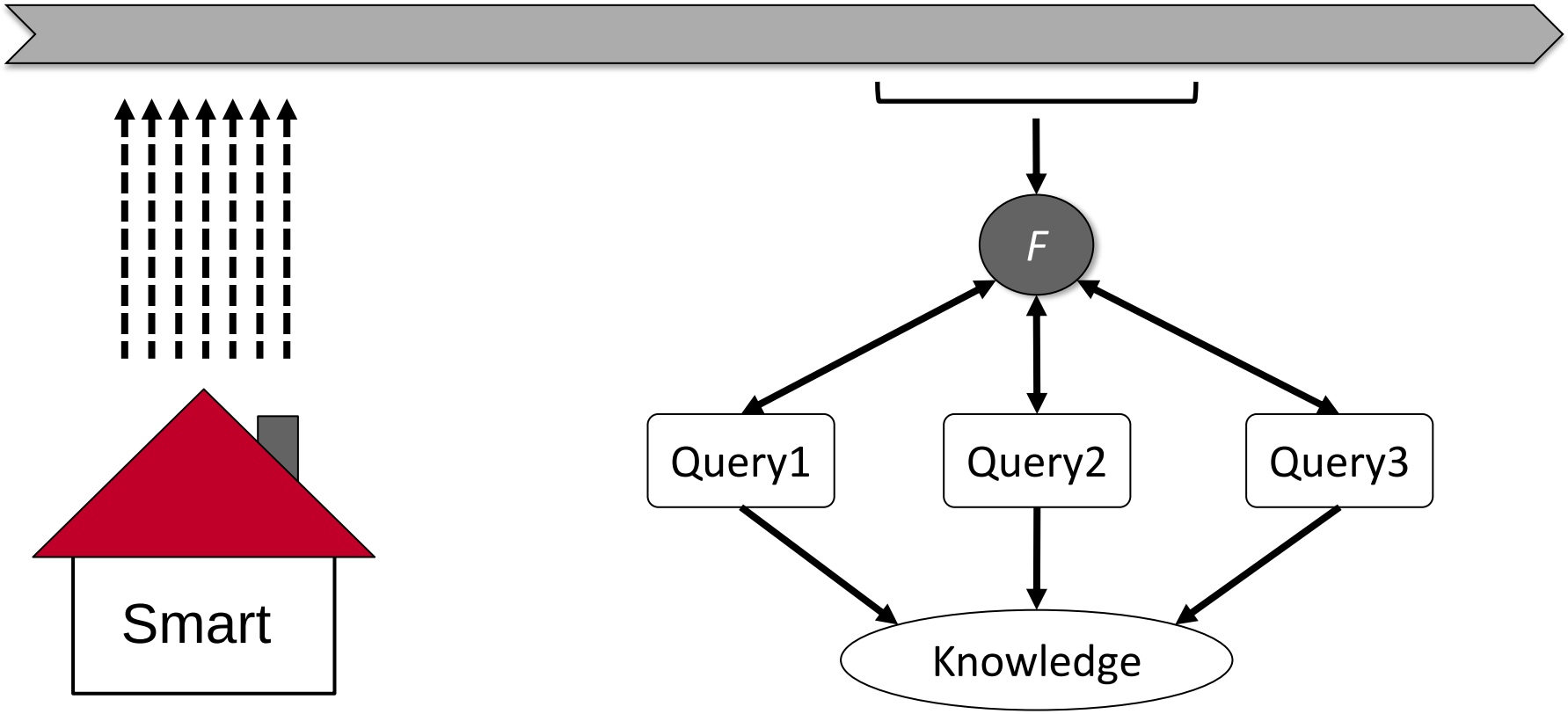
- energy supplier
- 1 million households

$\Rightarrow$ about 2 PBs of data

$\Rightarrow$ constant update of patterns


EDDO
f
Smart


EDDO
Inference
f
F
Smart

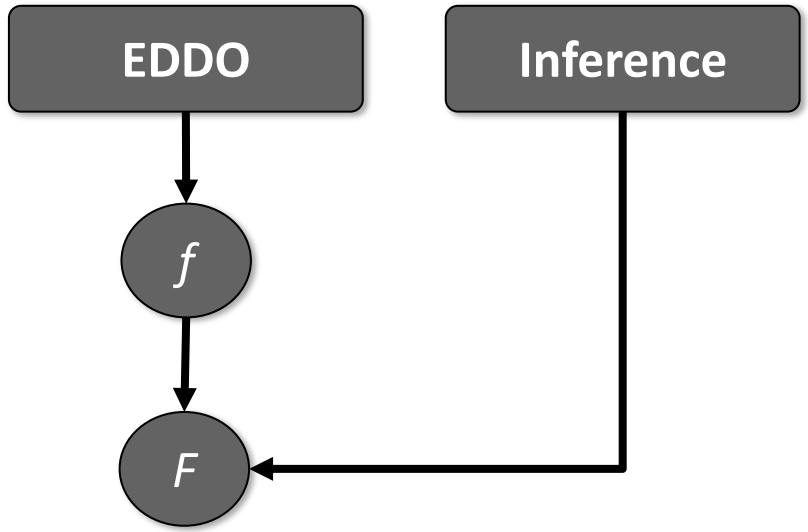


Query:

Return the probability distribution for sensors in the living room during the week days.

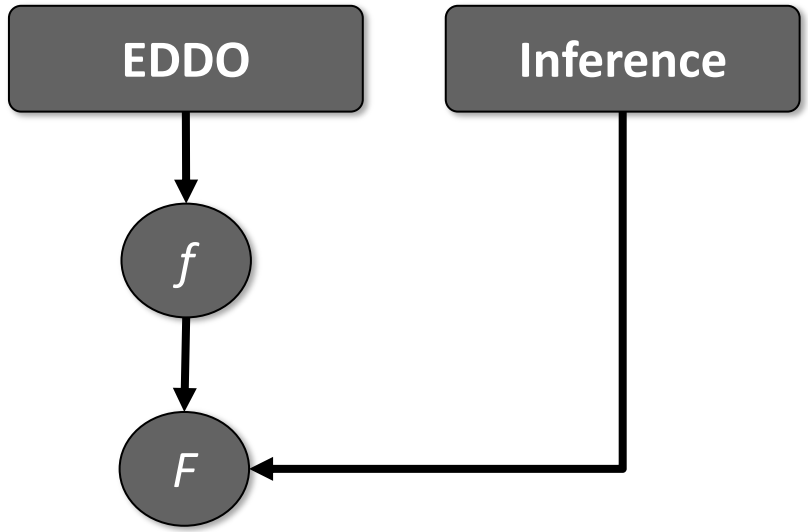
Weaknesses of EDDO

$$f(X_1, \ldots, X_n)$$



Weaknesses of EDDO

$$\mathrm{f}(X_1) \cdot \prod_{i=2}^{n} f(X_i | X_1, \ldots, X_{i-1})$$



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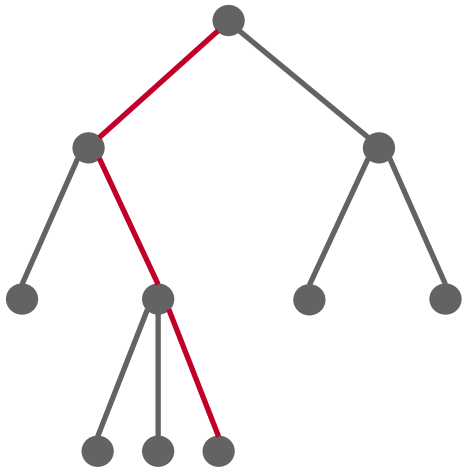

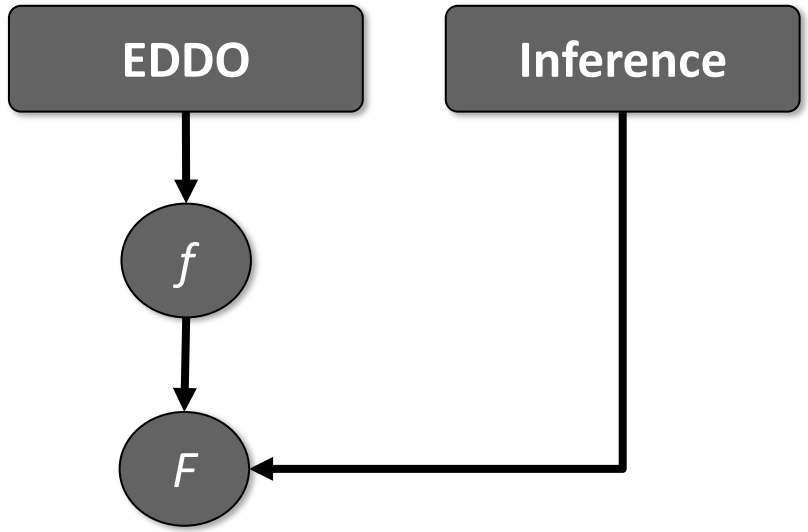

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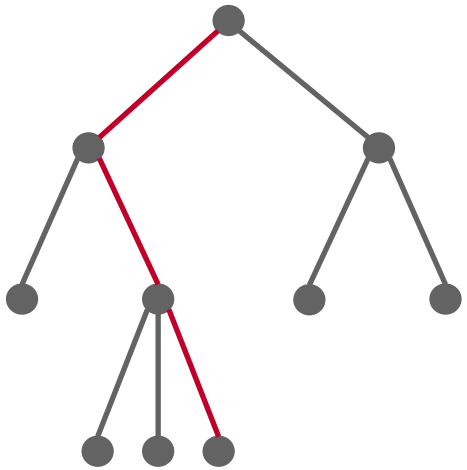
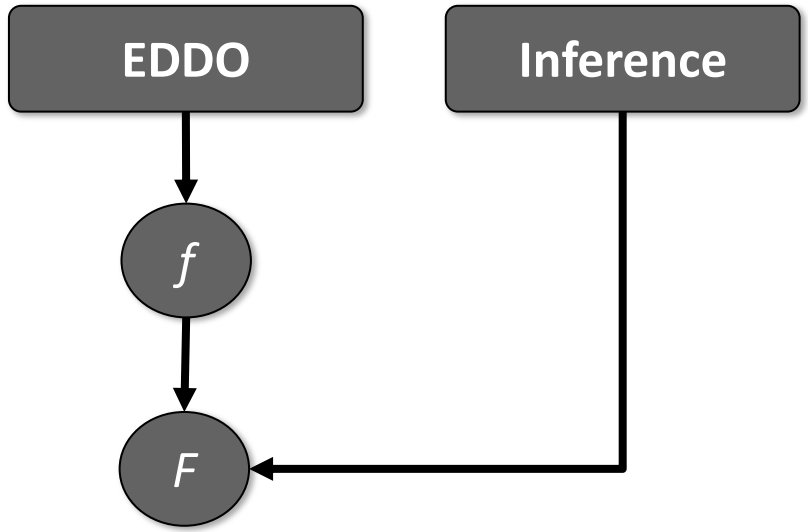




only for discrete random variables

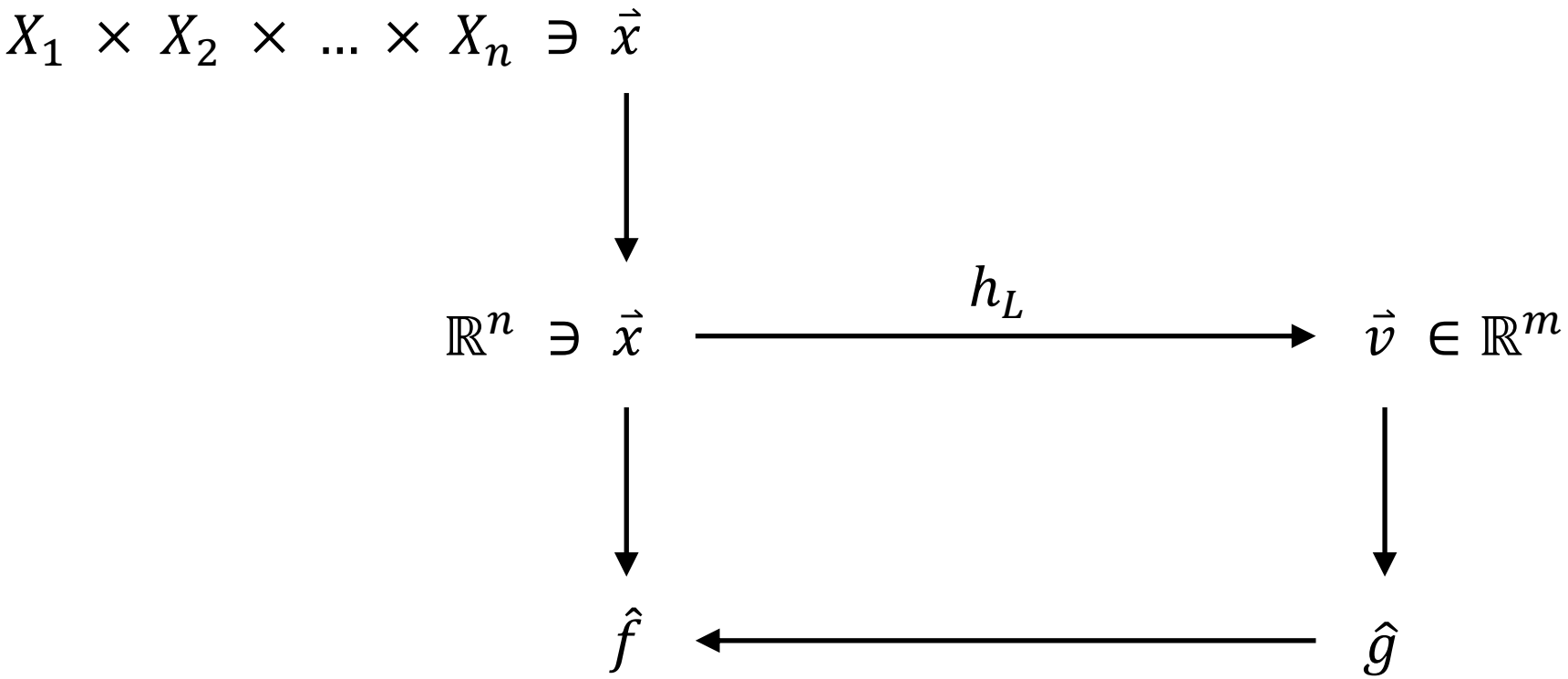
Goals

A density estimator that

- estimates joint densities from data streams
- is able to deal with heterogeneous data, and
- and works for higher dimensional data.

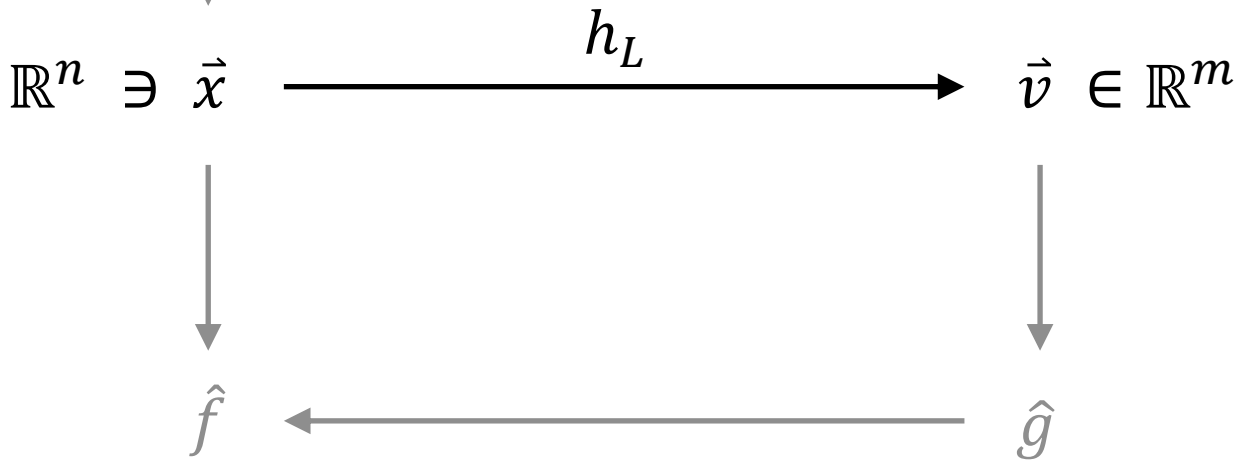
For density estimation, 100 variables is high dimensional.

Main Idea

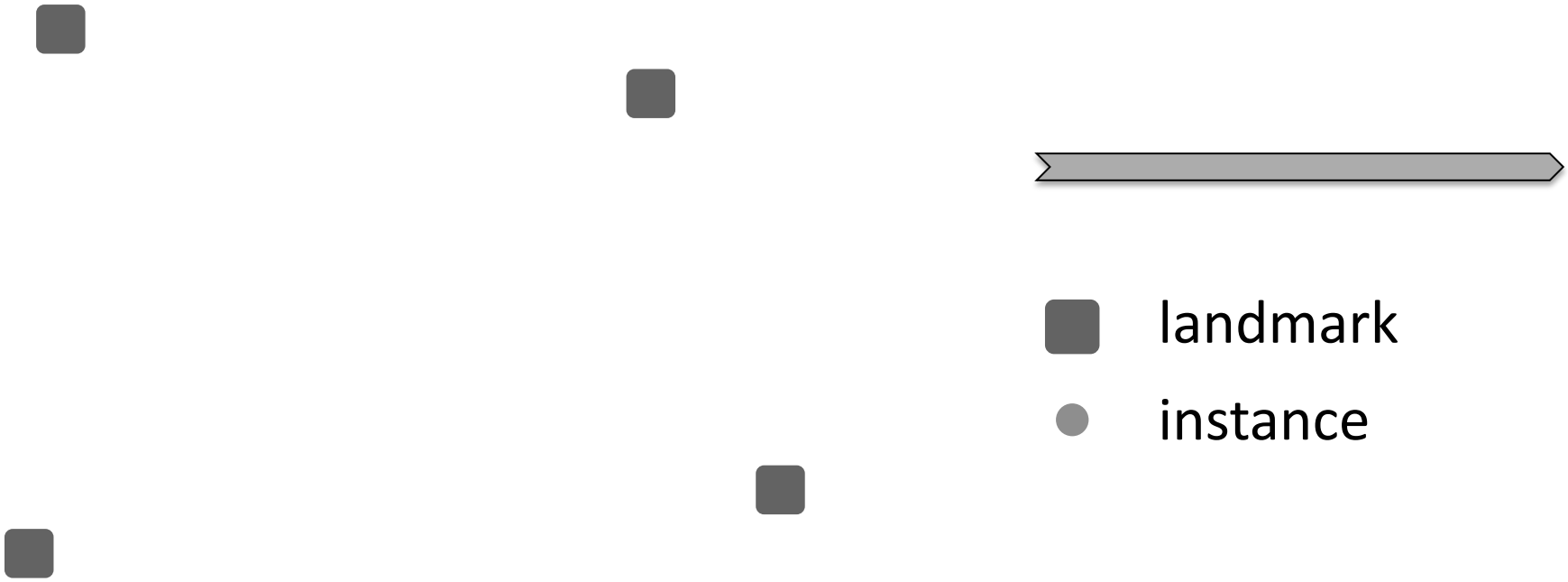

$X_1 \times X_2 \times \ldots \times X_n \ni \vec{x}$
$\mathbb{R}^n \ni \vec{x}$
h_L
$\vec{v} \in \mathbb{R}^m$
$\hat{f}$
$\hat{g}$

Main Idea

$X_1 \times X_2 \times \ldots \times X_n \ni \vec{x}$

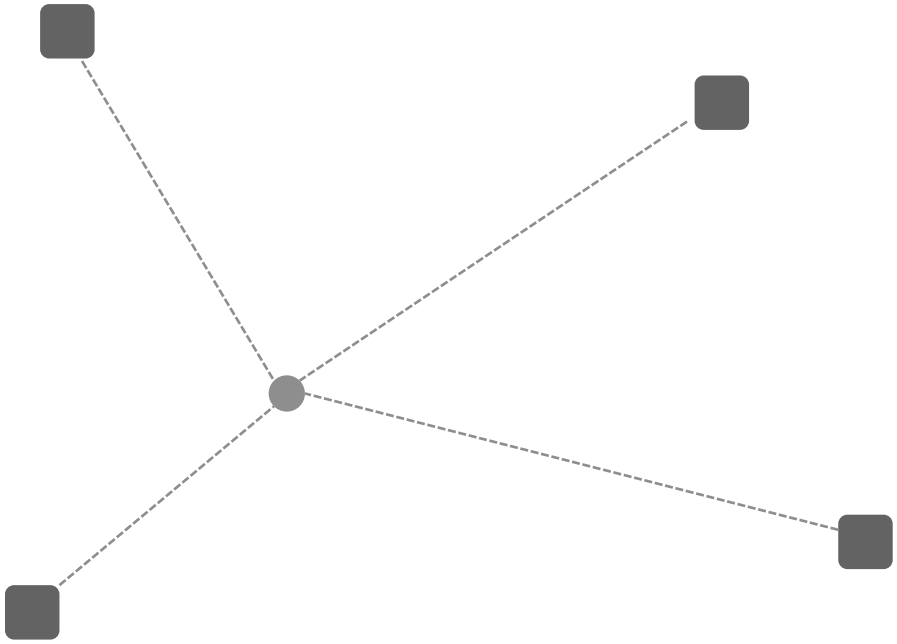


Online Density Estimation using Representatives (RED)



$(temperature = 20, humidity = 50, \dots) = \blacksquare \in \mathbb{R}^n$

Online Density Estimation using Representatives (RED)



landmark

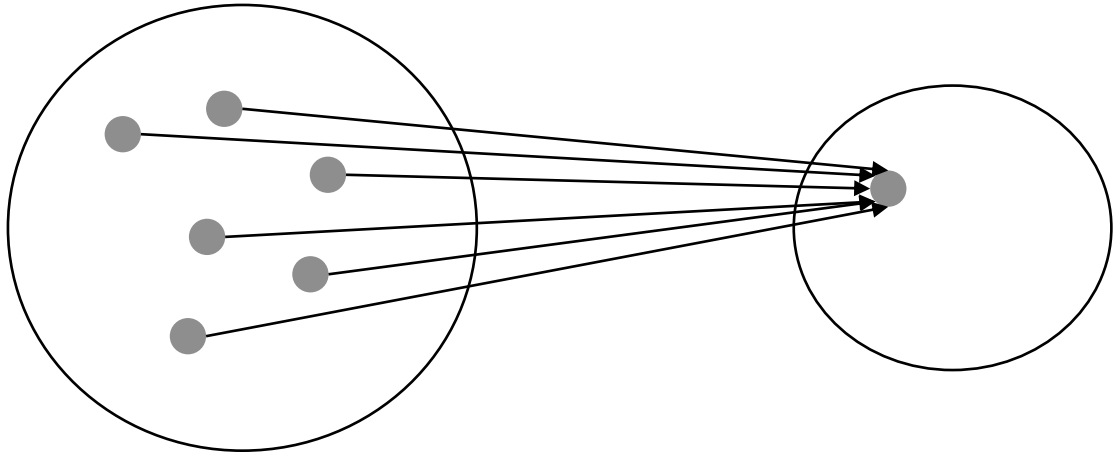
instance

$$\mathbb{R}^n \ni \vec{x} = (x_1, x_2, \ldots, x_n) \longrightarrow \vec{v} = (v_1, v_2, v_3, v_4) \in \mathbb{R}^m$$

Online Density Estimation using Representatives (RED)

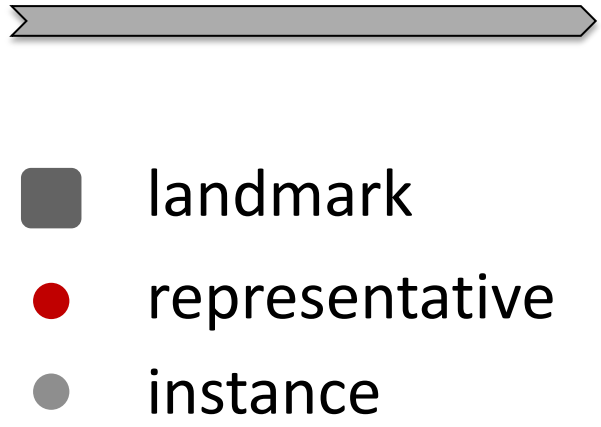
$$I = \{\vec{x} \in \mathbb{R}^n \mid h_L(\vec{x}) = \vec{v} = (v_1, v_2, v_3, v_4)\}$$

$$\hat{g}(\vec{v}) = \sum_{\vec{x} \in I} \hat{f}(\vec{x})$$



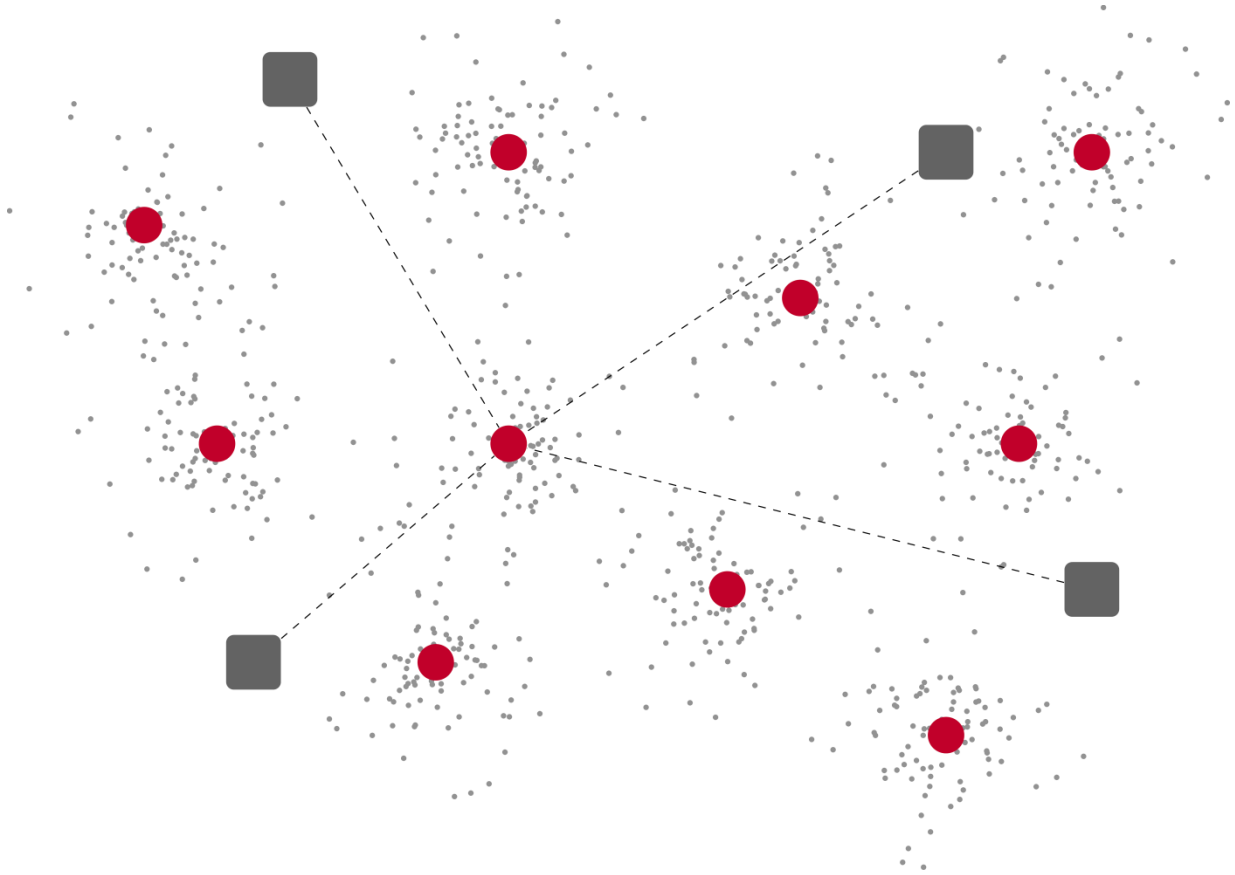
$\mathbb{R}^n$ $\mathbb{R}^m$

Online Density Estimation using Representatives (RED)



$(temperature = 10, humidity = 80, \dots) =$ ● $\in \mathbb{R}^n$

Online Density Estimation using Representatives (RED)



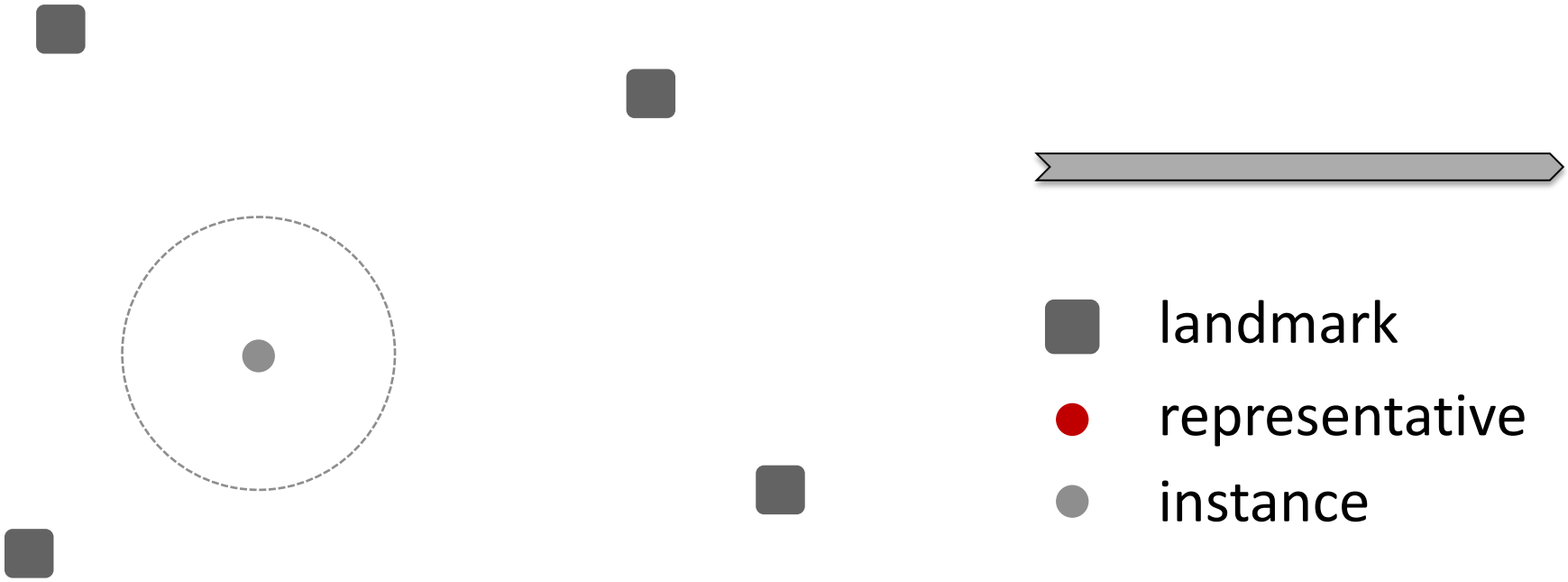

landmark


representative


instance

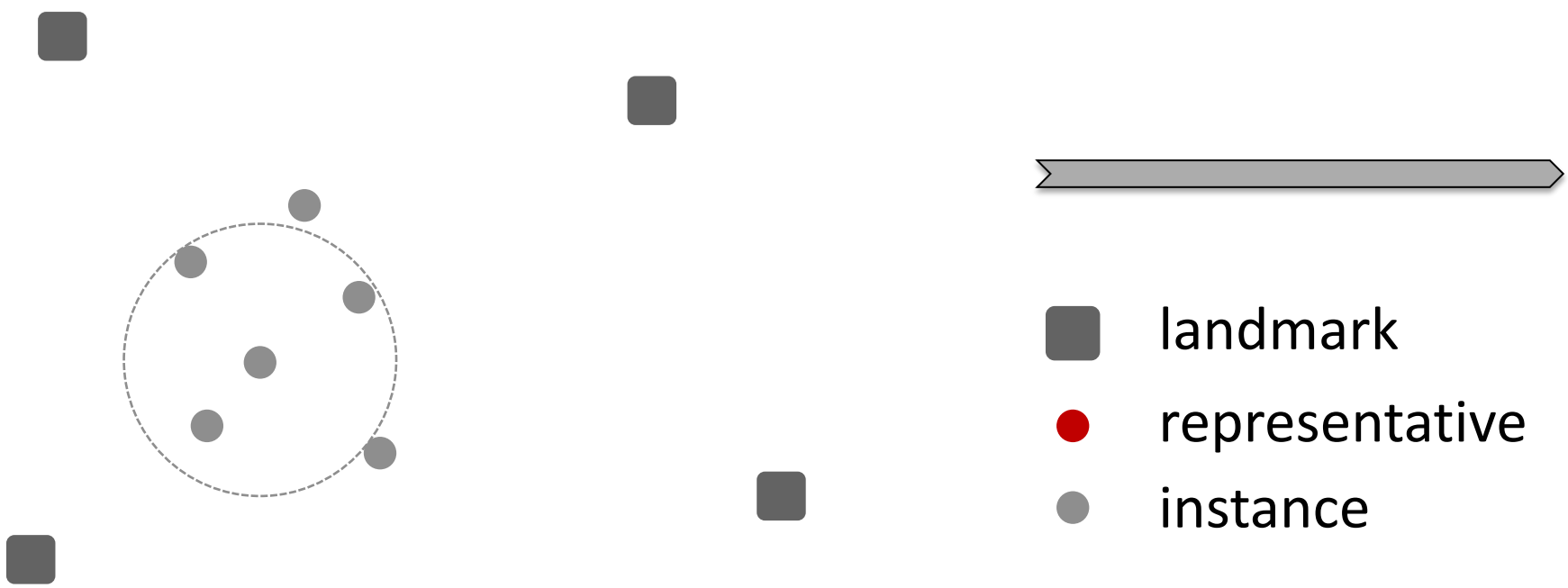
$$\{\vec{x} \in \mathbb{R}^n \mid h_L(\vec{x}) = (v_1, v_2, v_3, v_4)\}$$

Online Density Estimation using Representatives (RED)



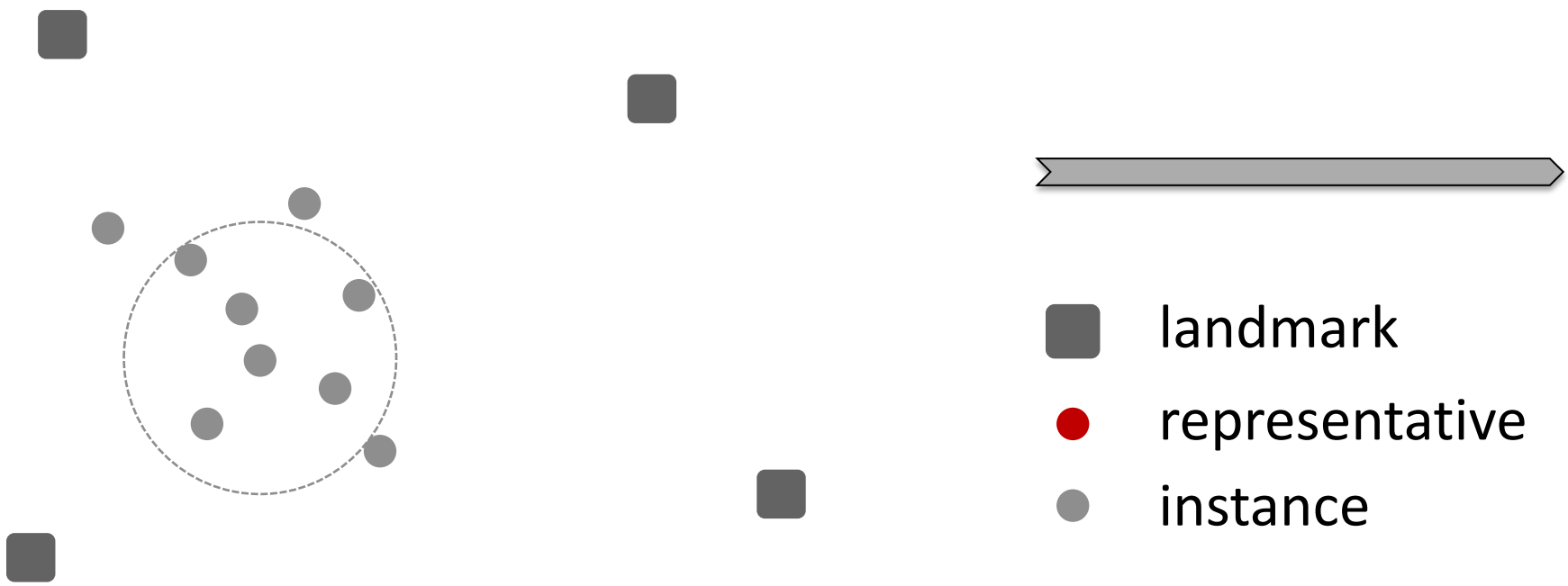
Mahalanobis distance: $\sqrt{(\vec{x} - \vec{v})^T \Sigma^{-1} (\vec{x} - \vec{v})}$

Online Density Estimation using Representatives (RED)



$$\vec{v} = (v_1, v_2, v_3, v_4)$$

Online Density Estimation using Representatives (RED)



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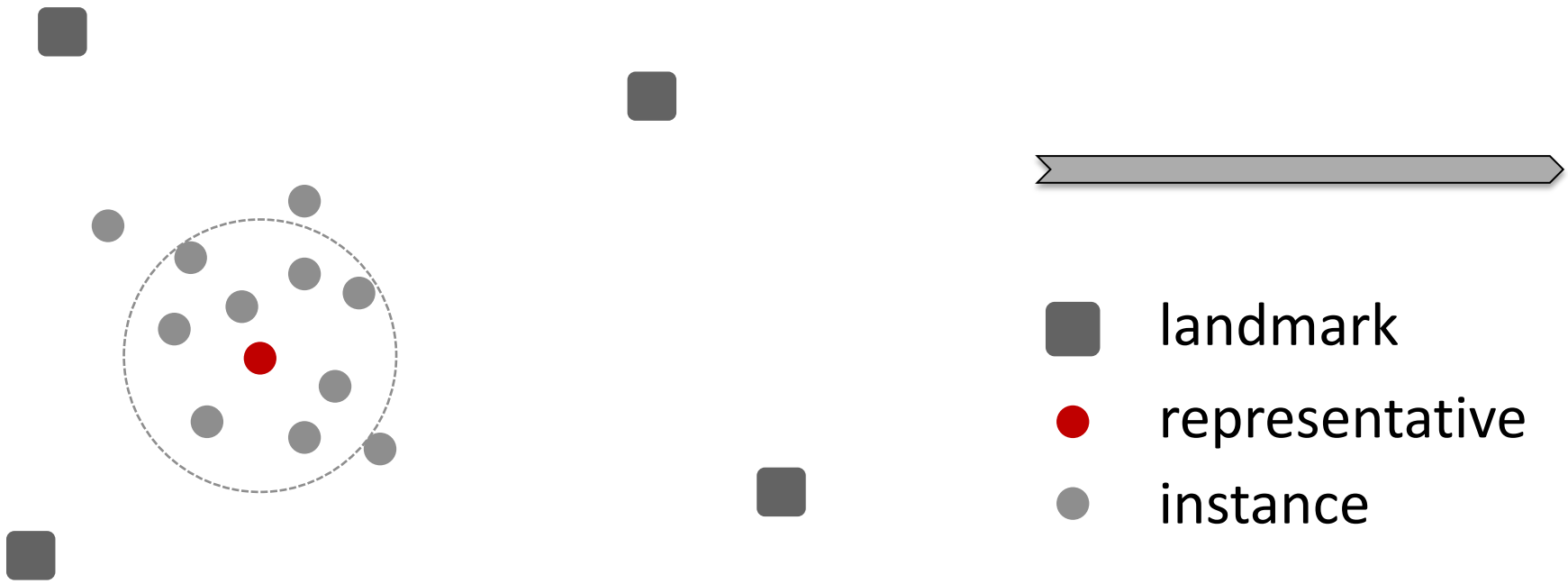
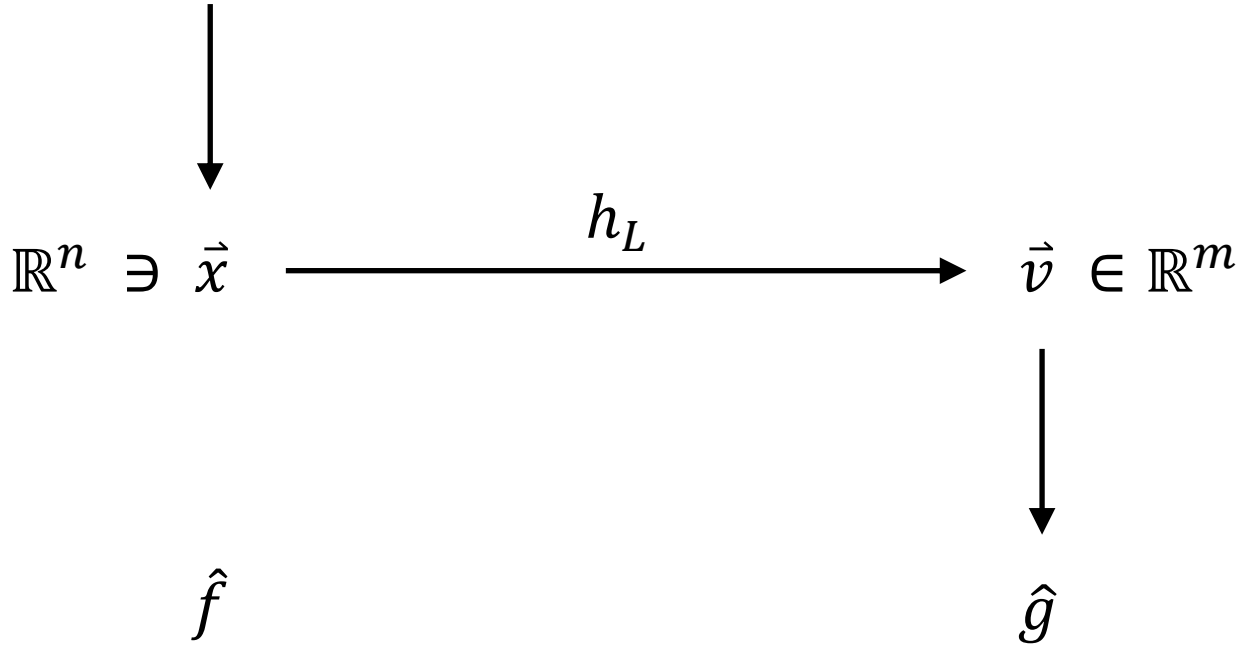


$$\vec{v} = (v_1, v_2, v_3, v_4) \longrightarrow \mathrm{f}(V_1) \cdot \prod_{i=2}^{m} f(V_i | V_1, \ldots, V_{i-1})$$

Online Density Estimation using Representatives (RED)

$X_1 \times X_2 \times \ldots \times X_n \ni \vec{x}$

$h_L(\vec{x}) = h_L(\vec{y})$ **but** $\vec{x} \neq \vec{y}$



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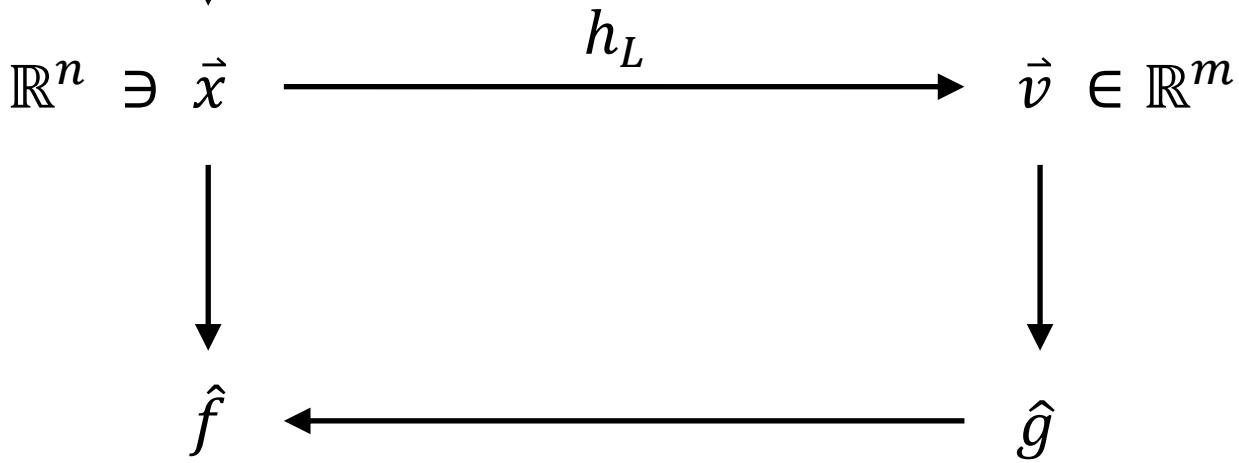


$\hat{g}(\vec{v}) = \sum_{\vec{x} \in I} \hat{f}(\vec{x})$

Choice of Landmarks

Main idea:

- theoretical foundation
- landmarks are orthogonal to each other
- if $|L|$= d + 1, then consistent estimator
- back translation by system of linear equations

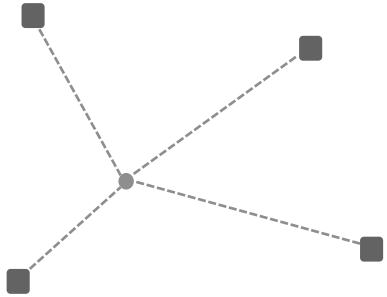
Evaluation: Parameter Setting

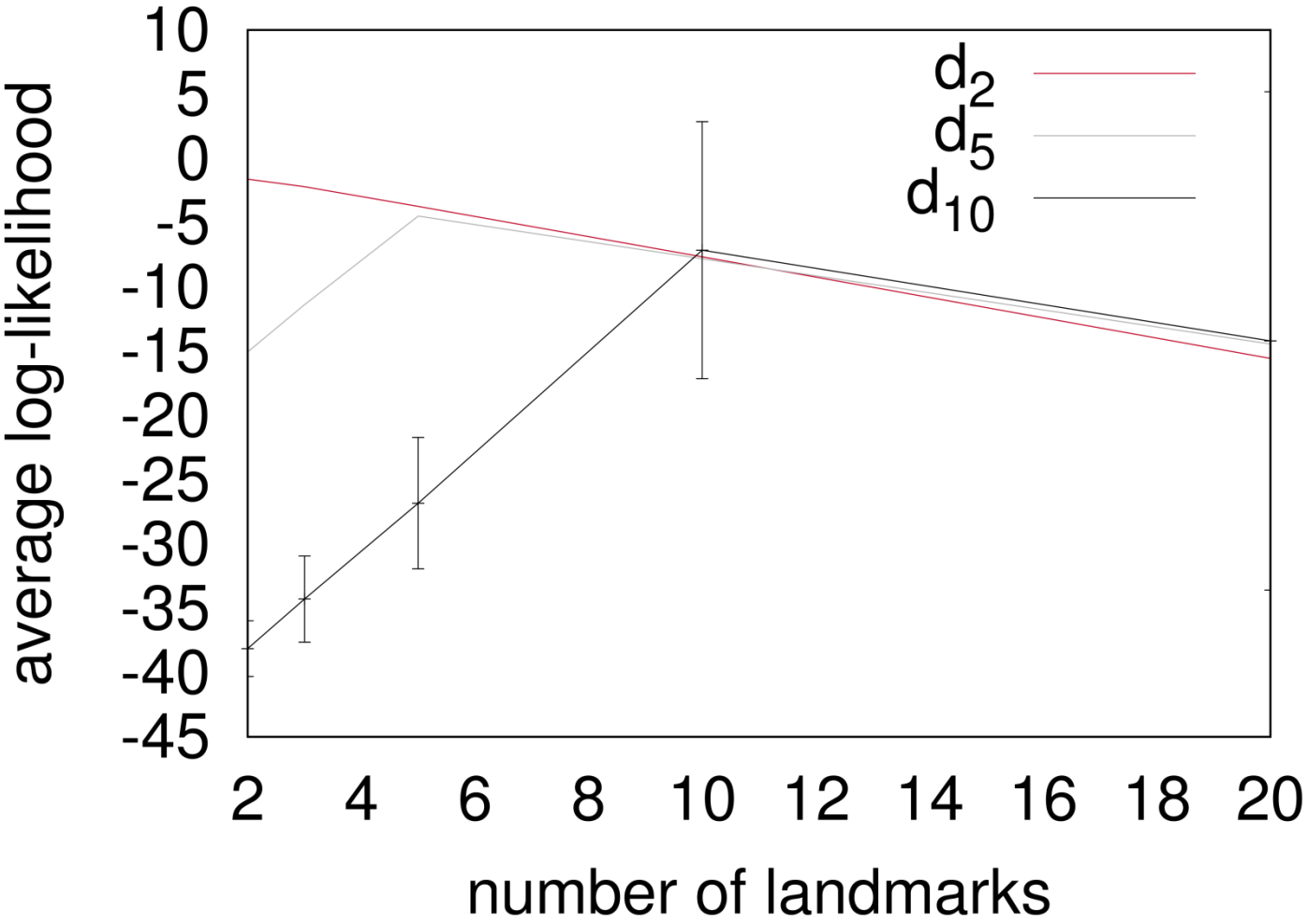
Parameters:

- $\theta_{C \to R} = 100$
- Euclidean norm
- $|L| \in \{2, 3, 5, 10, 20\}$
- $M \in \{0.1, 0.5, 1.0, 2.0, 5.0, 10.0\}$

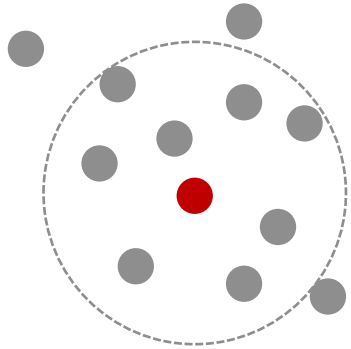
Datasets
Synthetic
Gaussian mixtures
Real-World
Covertype Electricity Letter Shuttle

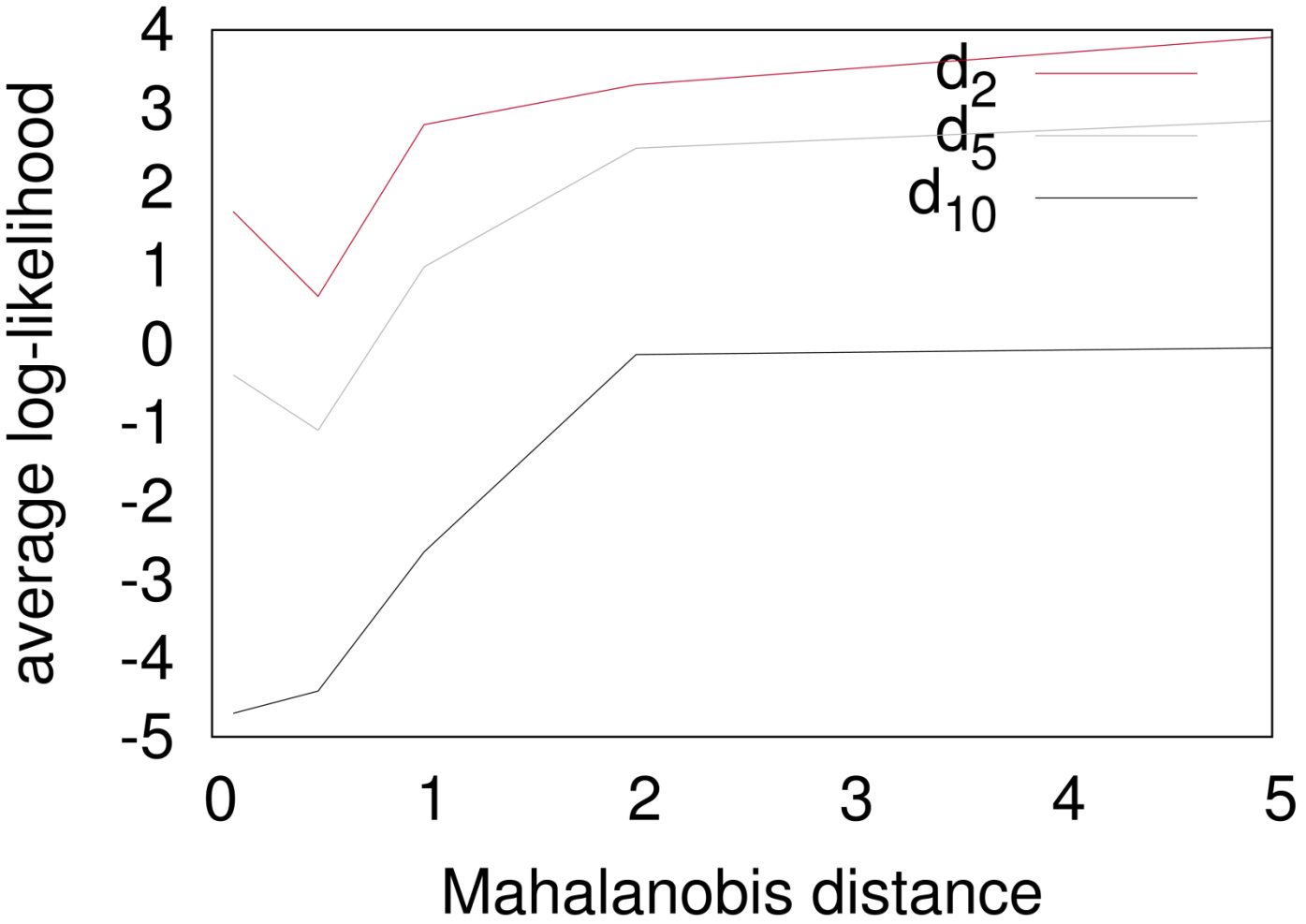
Evaluation: $|L|$



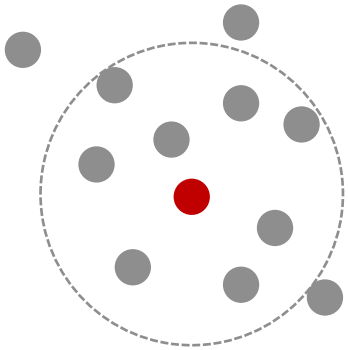


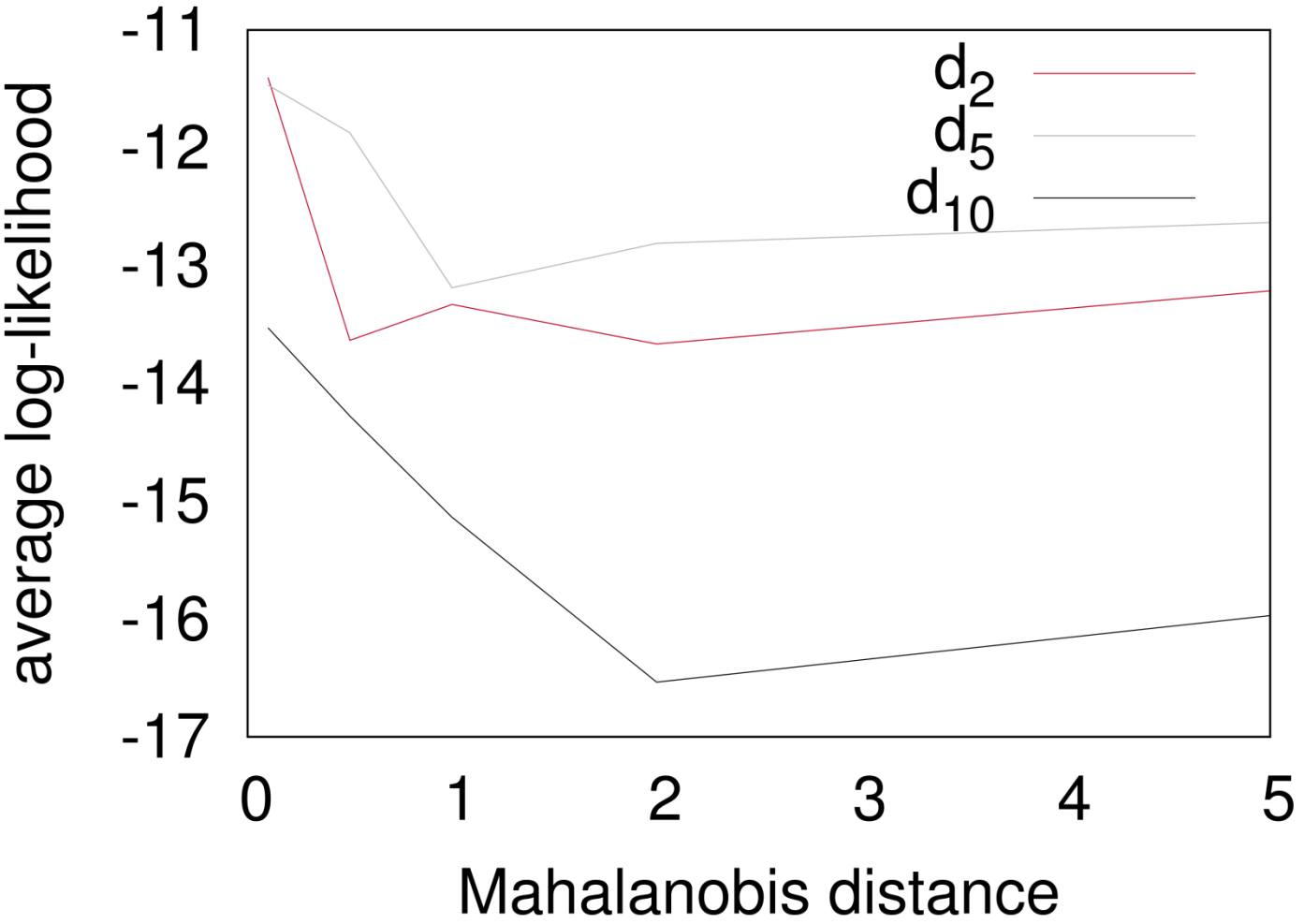
Evaluation: Mahalanobis (1 Gaussian)



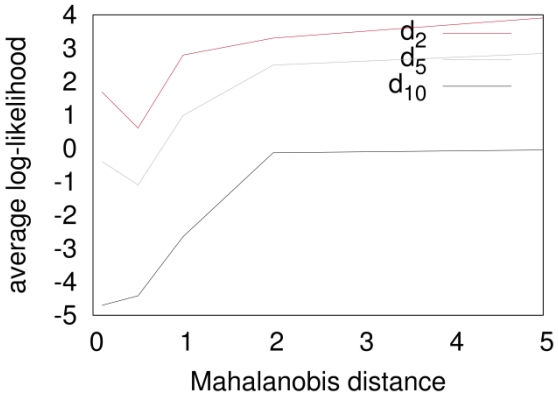


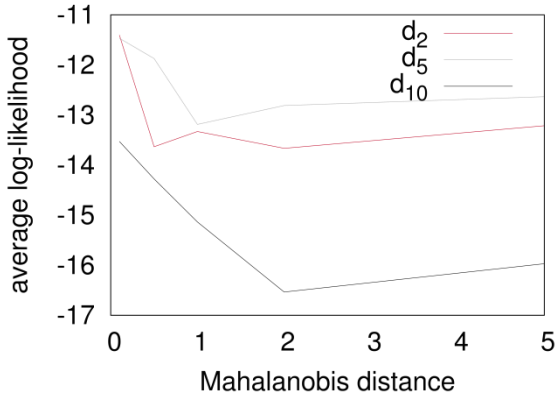
Evaluation: Mahalanobis (10 Gaussians)

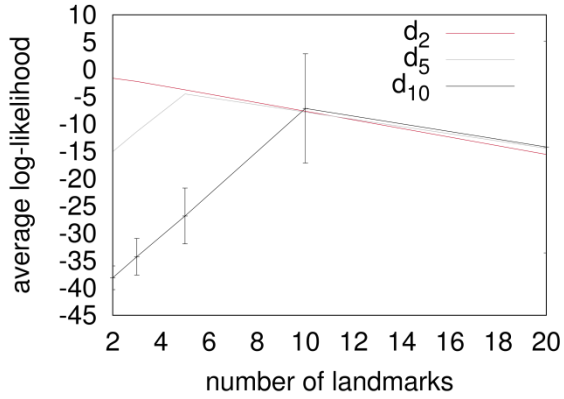




Evaluation: Parameter Setting







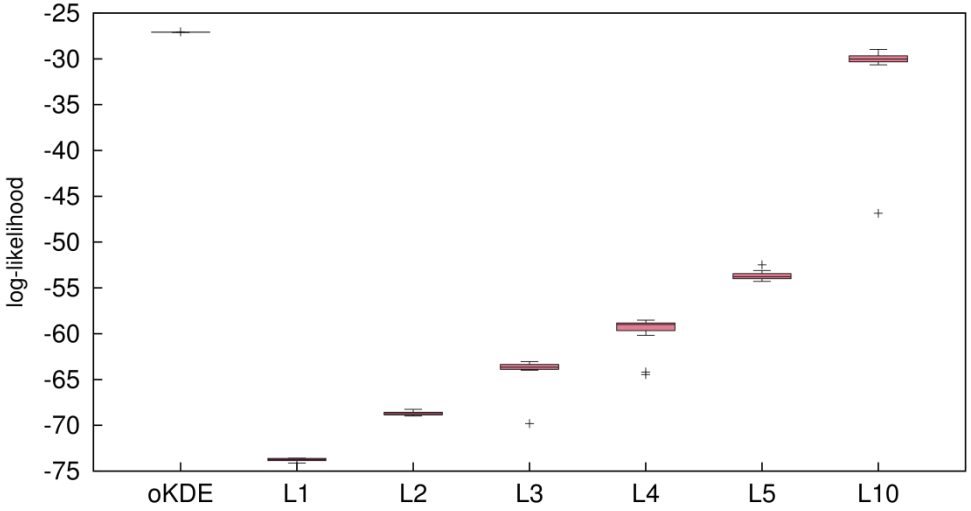
- $|L|$ depends on dimensionality of data
- small M partition the space better
- but at some point too few instances per region

Evaluation: Performance

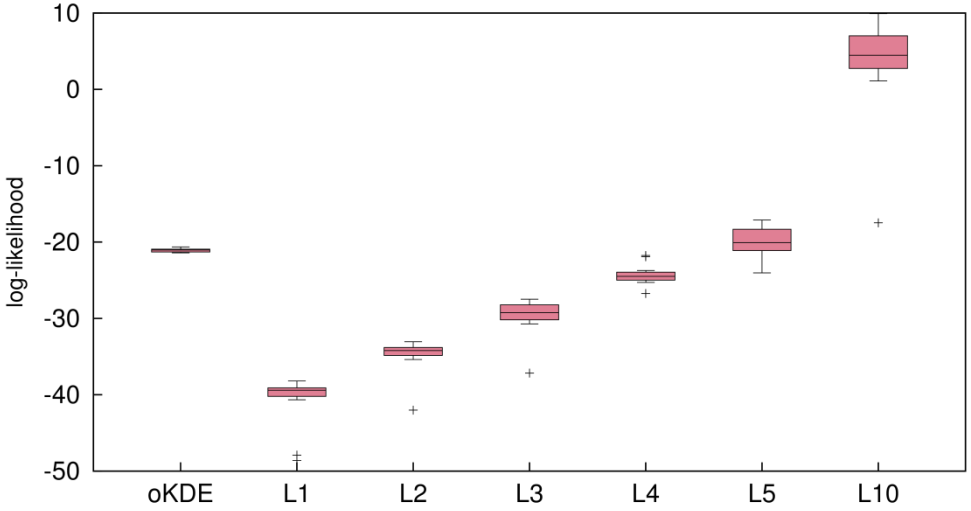
oKDE:

- online Kernel Density Estimator
- for multi-variate densities
- for continuous variables
- by Kristan et al. (2011)

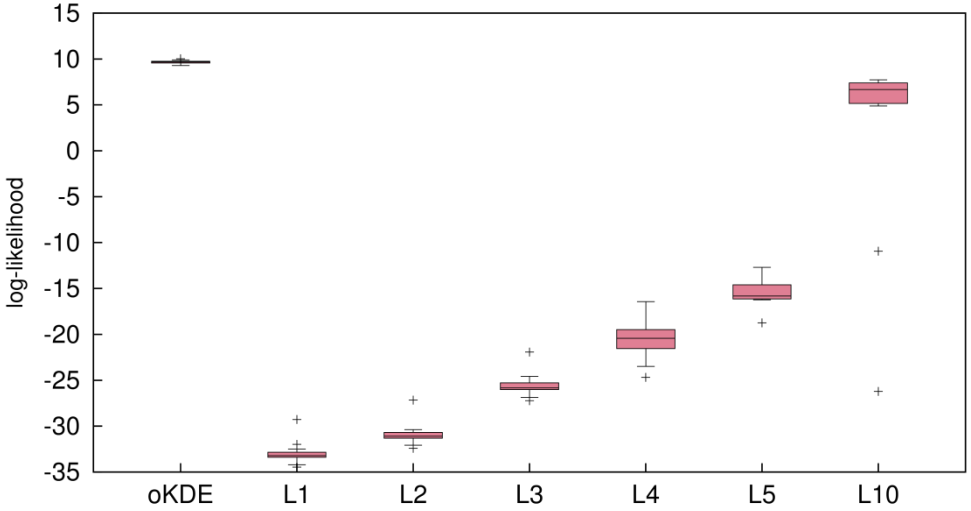
Datasets
Synthetic
Gaussian mixtures
Real-World
Covertype Electricity Letter Shuttle



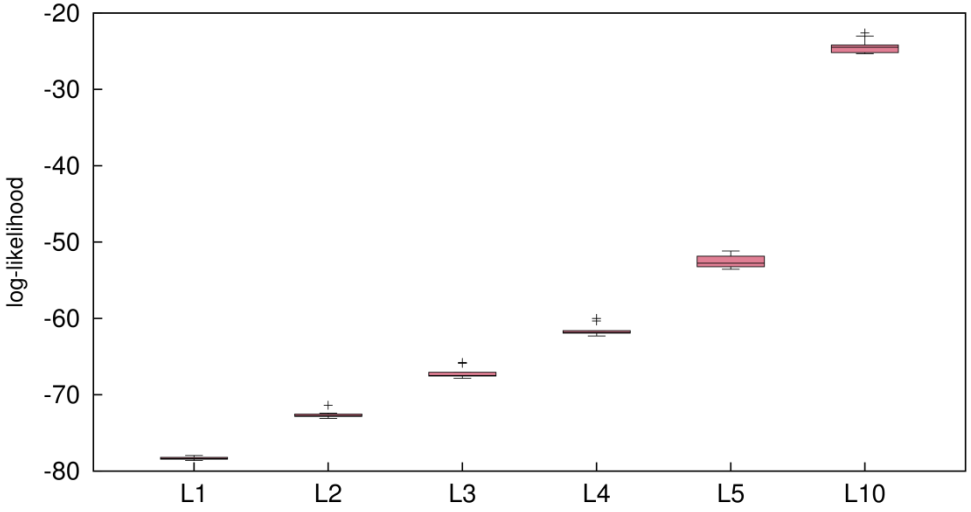
electricity (9 attributes)



shuttle (11 attributes)

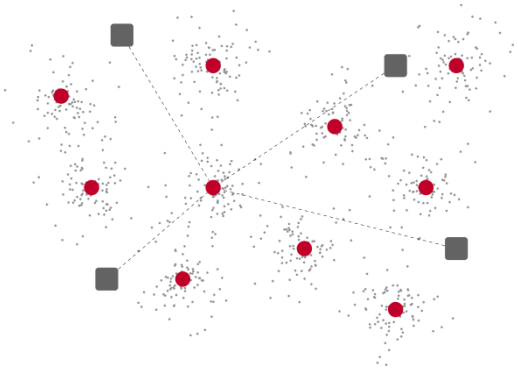


letter (17 attributes)



covertype (54 attributes)

Conclusions



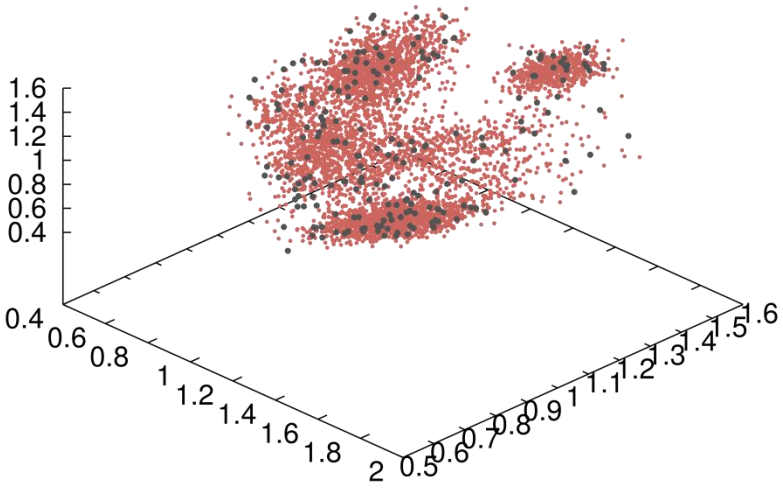
- online density estimation in higher dimensions
- heterogeneous data stream
- theoretical foundation
- comparable to the state of the art

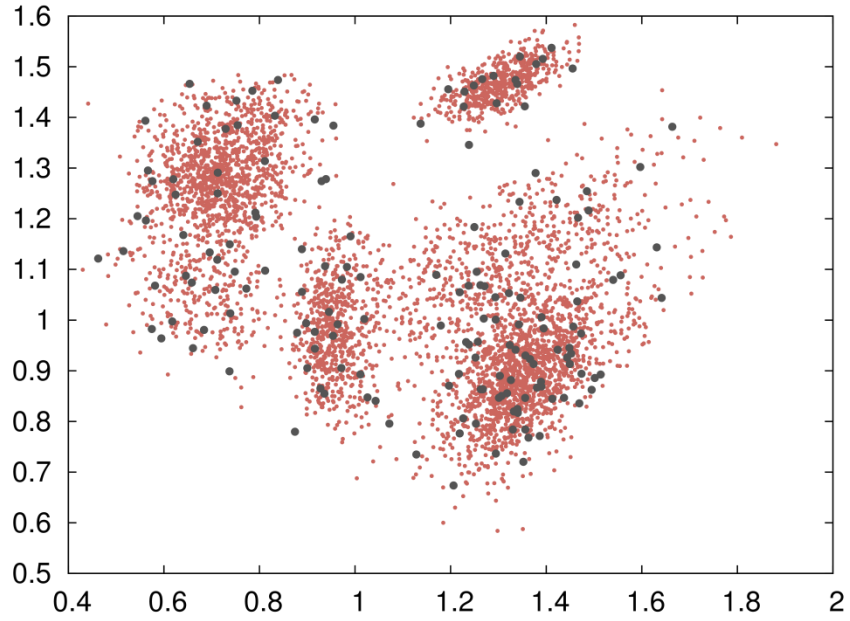
Future Work:

- new strategies for landmarks selection
- outlier detection
- detection of emerging trends

Thank you for your attention

Online Density Estimation using Representatives (RED)





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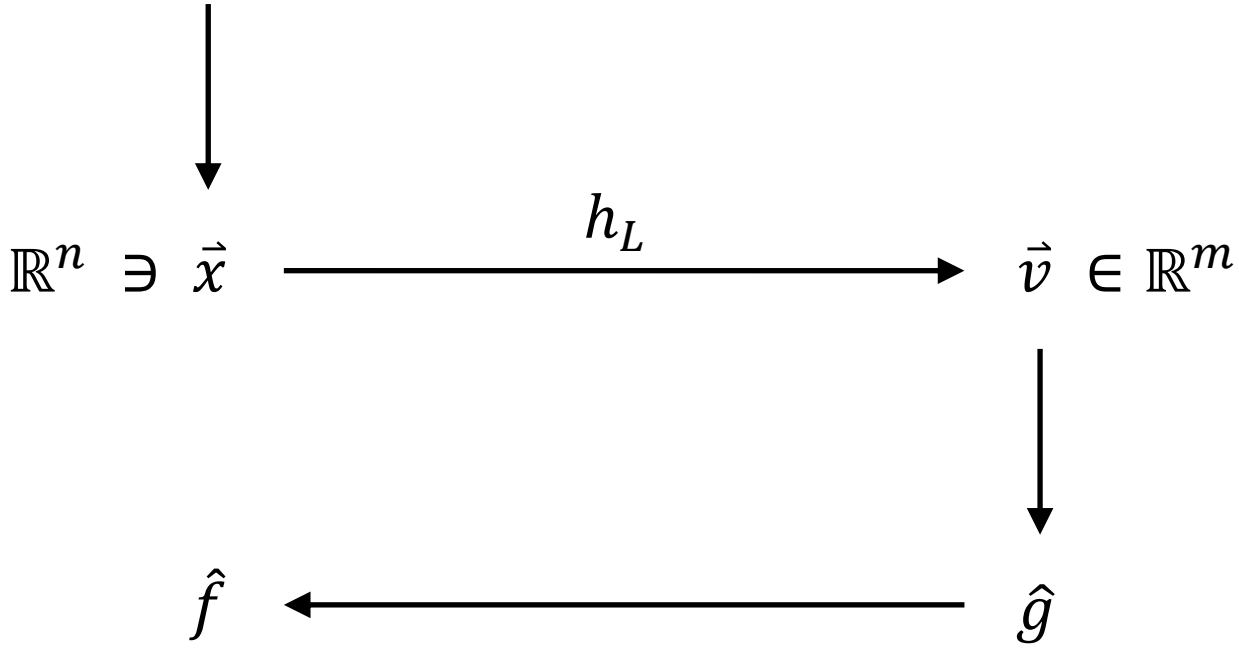
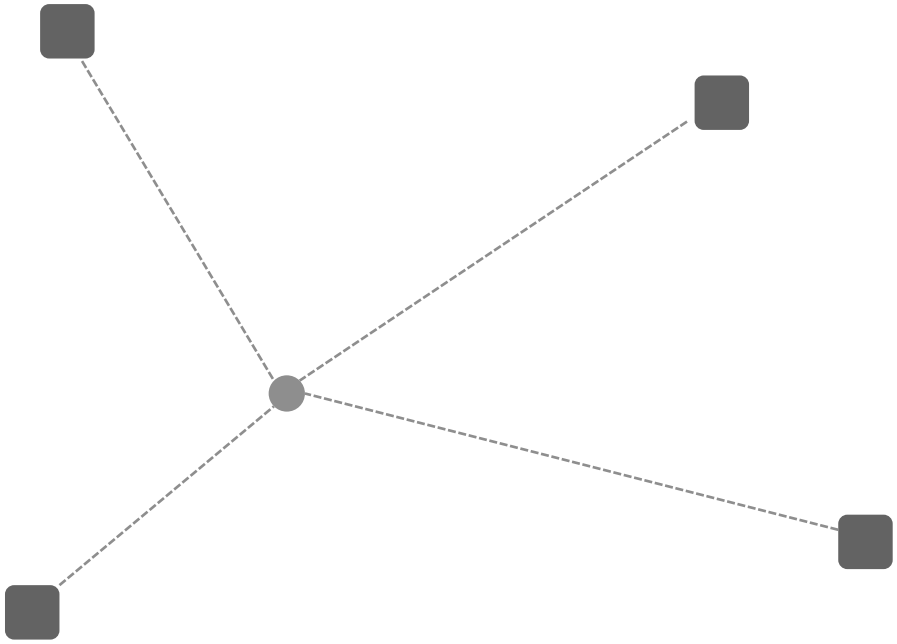


$$\iiint_{-\infty}^{-\infty} \prod_{j=i}^{p} corr_j\left(x_j \middle| v_1, \ldots, v_p\right) \, dx_{i+1} dx_{i+2} \ldots d_n$$

Online Density Estimation using Representatives (RED)



landmark

instance

$$I = \{\vec{x} \in \mathbb{R}^n \mid h_L(\vec{x}) = \vec{v} = (v_1, v_2, v_3, v_4)\}$$